A COMPARISON OF ALTERNATIVE NET PRESENT VALUE FORMULATIONS - IMPLICATIONS FOR DEBT CAPACITY

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by

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ABSTRACT

Two widely-used net present value formulas, the weighted average cost of capital formula and the return to equity formula are reconciled for both a single period and a multiperiod case. In both cases, the differences in NPVs emerging from the two formulas can be attributed to alternative assumptions about debt capacity.

Introduction

The net present value method of investment analysis is widely endorsed by agricultural economists both in textbook exposition of capital budgeting theory and in research on the determinants of capital expenditures by farm and agribusiness firms. While there is little disagreement as to the value of this method, there are some significant differences in the way in which variables are defined for the analyses. In particular, opinion differs on how best to account for the role of financing in the capital budgeting model. Few would assert that this role is unimportant. At issue, however, is the proper manner in which to convey it.

One major approach, termed the return to equity (RTE) approach calls for adjusting the cash flows to reflect financing. The other approach, termed the weighted average cost of capital (WACC) approach calls for an adjustment to the discount rate. These approaches usually result in different net present values for identically specified investments. The purpose of this paper is to examine the conditions under which the approaches can be reconciled.

Defining the Alternatives

The RTE approach is explained and illustrated in textbooks by Barry, Hopkin, and Baker and Penson and Lins. It also forms the basis for most farmland bid price models. The distinguishing feature of this approach is the inclusion in the cash flow budgets of principal and interest payments on the particular loan used, or expected to be used to finance the investment in question.

Algebraically, the approach is expressed as:

$$NPV = \sum_{t=1}^{T} \frac{(C(t)-r(t)D(t))(1-\tau)-D(t)}{(1+k_e)} - E_o \qquad (1)$$

Where:

$C(t)$ = cash flow before interest and taxes in period t.
$r(t)$ = interest rate in period t.
τ = marginal tax rate on ordinary income.
$D(t)$ = principal paid in period t.
k_e = discount rate used to capitalize cash returns to equity.
E_o = initial equity investment (downpayment) in the project.

The net cash flow described by the equation above represents cash available to the equity owner. The equity discount rate, k_e represents the equity owner's "hurdle rate", a minimum rate of return, after taxes, that must be earned on equity committed to an investment with given risk characteristics. The debt capacity assigned to the investment in any period t will equal the value of D(t).

The WACC approach, described by Lee, et al. and by Casler, Anderson, and Aplin, uses a weighted average discount rate to reflect the relative contributions of debt and equity, respectively, to the investment. The cash flows developed for this approach do not reflect the deduction of interest or principal payments. However, the net present value emerging from this formulation represents a return to equity capital just as in the RTE approach. These two approaches are thus intended to measure the same thing, the net contribution of the particular investment project to the market value of the firm's equity.

Algebraically, the weighted average cost of capital approach is expressed as:

$$NPV = \sum_{t=1}^{T} \frac{C(t)(1-\tau)}{(1+k_w)} - I_o \qquad (2)$$

Where:

$C(t)$ = cash flow before interest and taxes in period t.
τ = marginal tax rate on ordinary income.
k_w = the firm's weighted average cost of capital.
I_o = the initial total investment in the project.

The value of k_w is generally deemed to reflect the marginal costs of debt and equity capital weighted by their anticipated market value proportions in the firm's optimal or desired capital structure (Lee, et al., p. 75) Accordingly,

$$k_w = w_d\, k_d + w_e\, k_e,$$

Where:

k_d = the after tax cost of debt capital.
k_e = the cost of equity capital.
w_d and w_e = the proportions of debt and equity capital, respectively, in the optimal or desired capital structure of the firm.

Under the WACC concept, the debt capacity of any project, regardless of how it is actually financed, is assumed to be given by the leverage proportion, w_d. If, for example, w_d = .50, then all investments are evaluated as if they were to be financed with 50 percent debt. To determine the dollar amount of debt capacity accruing to an incremental investment, the proportion w_d is multiplied by the market value of the incremental investment. A problem arises, however, in that there are two market value concepts to consider. The first, termed by Copeland and Weston (p.279), the replacement value, is the economic cost of putting the project into place, or ΔI. The second, termed the reproduction value, is the total present value of the stream of cash flows expected from the project, or ΔV. The relationship between these two concepts is the net present value of the investment; that is,

$$NPV = \Delta V - \Delta I.$$

Thus, to the extent that the reproduction value exceeds the replacement value, net present value is positive and the firm's debt capacity, in dollar terms, will have increased as a result of making the investment. If a $1,000 incremental investment yields a present value of \$1,300, its net present value is \$300. Assuming that w_d = .50, the dollar debt capacity based on replacement value is \$500. Based on reproduction value, the debt capacity is \$650.

Most writers, while agreeing that the ratio expressed by w_d should be the ratio D/V, also agree that the dollar debt capacity of an incremental investment should be expressed as $w_d(\Delta I)$ (Beranek). In the example just given, the investment analysis would proceed on the assumption that the dollar debt capacity of the incremental investment is w_d (ΔI) = \$500. The fact that the NPV of the investment is \$300 indicates that by undertaking the investment, the firm is increasing its debt capacity by $w_d(\Delta V - \Delta I)$ = \$150.

This issue emerges as an important one in the comparison of the WACC and RTE approaches. Procedurally, the RTE approach defines the debt capacity of an incremental investment in terms of a dollar amount (i.e., ΔD) while the WACC approach defines it in terms of a ratio, w_d. Under the WACC approach, debt capacity will be a constant proportion regardless of whether market value is defined as ΔV or ΔI. Under the RTE approach, the percentage debt capacity of an incremental investment will differ depending on which definition of market value is used. That is, the ratio $\Delta D/\Delta I$ will not equal $\Delta D/\Delta V$ except in the special case where NPV = 0.

Reconciling the Alternatives: The One-Period Investment

The two approaches can be reconciled by equating the explicit weighted average cost of capital employed in the WACC approach with that implied by the RTE approach. The WACC approach establishes a weighted average cost of capital via an ex ante rule (i.e., k_w is the value that conforms to the optimal or desired capital structure). The RTE approach can yield an implied weighted average cost of capital but it does so ex post. However, it is valid to compare the two because both approaches express the same objective function, the maximization of equity-owners wealth.

Algebraically, the reconciliation is achieved by setting equation (1) above equal to equation (2) and solving for k_w. Assuming that the firm consists of a single, one-period investment and dropping the time subscripts;

$$\frac{(c-rD)(1-\tau)-D}{1+k_e} - E = \frac{C(1-\tau)}{1+k_w} - I. \qquad (3)$$

Setting D = I - E and rearranging, the value of k_w, which will be designated k_w', is:

$$k_w' = \frac{C(1-\tau)}{\dfrac{C(1-\tau) + rD\tau - D(1+r)}{1+k_e} + D} - 1 \qquad (4)$$

The numerator in this expression is the after-tax cash flow before financing costs. The denominator is the total present value of the cash flows resulting from the incremental investment, or ΔV. This consists of the present value of equity, which is the sum of the after-tax cash flow plus the tax shield on interest minus the debt principal and interest payment,

$$\frac{C(1-\tau) + rD\tau - D(1+r)}{1+k_e}$$

and the present value of debt,

$$D = \frac{D(1+r)}{(1+r)}$$

where the market value equals the sum of principal and interest payments discounted by the debt holder's discount rate (which equals r, the interest rate charged on the debt).

The value of k_w is that which equates the net present value under the WACC approach with the net present value of the RTE approach. The comparison of k_w' with k_w will be made with the use of a numerical example.

Assume that a firm consists of a single investment project which requires an outlay, ΔI, of \$300 at time t=0 and yields a cash flow of \$500 at time t=1. The investment will be financed with \$150 in equity and a loan of \$150 repayable with interest at 10 percent at t=1 (assume w_d = .5). The equity capitalization rate is assumed to be 20 percent and the marginal income tax rate, 30 percent. The net present value of this investment via the RTE approach is;

$$NPV(R) = \frac{(\$500-\$15)(1-.3) - \$150}{(1+.2)} - \$150$$

$$NPV(R) = \$7.92$$

This calculation implies, according to equation (4), a weighted average cost of capital of;

$$k_w' = \frac{\$350}{\dfrac{\$350+\$4.5-\$165}{1.20} + \$150} - 1$$

$$k_w' = 0.1367.$$

Substituting k_w = .1367 for k_w in the WACC formula and solving for NPV(W);

$$NPV(W) = \frac{\$500-\$150}{1.1367} - \$300,$$

$$NPV(W) = \$7.91.$$

Note especially that the two net present values can be reconciled only if the debt capacity reflected in the WACC approach is given by $\Delta D/\Delta V$. To show this, calculate k_w by first using $w_d = .50$,

$$k_w \; .10(1-.3)(.5) + .20(.5)$$

$$k_w = 0.135,$$

and second by using $w_d = \Delta D/\Delta V$, where $\Delta D = w_d \; (\Delta I)$,

$$k_w = .10(1-.3)\left(\frac{\$150.00}{\$307.92}\right) = .20\left(\frac{\$150.00}{\$307.95}\right)$$

$$k_w = 0.1367.$$

Thus, the weighted average cost of capital implied by the RTE approach, $k_w^{'}$, is based upon a marginal debt capacity given by the ratio $\Delta D/\Delta V$ where ΔD is the dollar amount of the incremental investment financed by debt.

It can be shown by an extension of the numerical example that as the investment's net present value grows due to a larger net cash flow, the dollar gap between NPV(R) and NPV(W) widens. This occurs because for a given ΔD, the difference between $\Delta D/\Delta I$ and $\Delta D/\Delta V$ widens as the net present value increases. As illustrated in Table 1 below, when net present value is positive, NPV(W) will always exceed NPV(R) and when net present value is negative, NPV(R) will always be less negative than NPV(W). The ratio of NPV(W) to NPV(R) is established by the ratio $1+k_e/1+k_w$ and will not change with a change in a net cash flow. However, it will vary with a change in w_d as shown in Table 1.

The Multiperiod Investment

In the multiperiod case, reconciliation of the RTE and WACC approaches requires that $k_w = k_w^{'}$ for each period. Over the life of an investment, the actual proportion of funding from alternative sources may change as the debt is amortized and the relative value of the debt and equity contributions to the project change.

Table 1. A Comparison of NPV(W) and NPV(R) at Different Levels of Net Cash Flow and Different Values of w_d*

Net Cash Flow	$w_d = .50$		$w_d = .75$	
	NPV(W)	NPV(R)	NPV(W)	NPV(R)
\$200.00	-\$177.00	-\$167.00	-\$173.00	-\$159.00
400.00	- 53.00	- 50.00	- 46.00	- 42.00
500.00	8.37	7.92	17.46	16.04
600.00	70.00	66.00	81.00	74.00
800.00	193.00	183.00	208.00	91.00

*The ratio NPV(W)/NPV(R) is 1.0568 and 1.0885, respectively for $w_d = .50$ and $w_d = .75$.

The WACC approach, as usually employed, implies that these contributions remain constant in proportion throughout the life of the investment. The fact that the debt for a particular investment is amortized in a certain manner is assumed not to influence the firm's total cost of capital. Implicit in this reasoning is the assumption that the proportion of debt to equity used in the weighted average cost of capital calculation represents an optimum. This optimum, which may be considered to reflect the minimum cost of capital, may also be considered, in the context of fixed initial equity, to reflect the optimal allocation of credit to borrowing and liquidity reserves, respectively.

In either case, the weighted cost of capital represents an opportunity cost. The firm that departs from its optimal allocation by using more debt than it desires suffers a loss in liquidity value that more than compensates for the additional value contributed by the debt. The firm using less than the desired amount of debt accrues value in the form of an improved liquidity position although this value is less than the marginal value of additional debt.

Under the RTF approach, debt capacity for an investment project is determined by the amortization schedule used in calculating cash flows. The implications for k_w' over the life of the investment can be seen from a numerical example.

Assume again that a firm consists of a single investment project but that the investment will last for five periods. Other characteristics of the investment are given below;

Investment outlay	$9,000
Annual cash flow before interest principal and taxes	5,000
Marginal tax rate	30%
Cost of equity capital	20%
Cost of debt capital	10%

Further assume that the firm will finance this project with a loan of $4,500 payable in equal principal payments with the interest calculated on the unpaid balance. The cash flows for the project are given below;

Table 2. Cash Flows for Multiperiod Investment Analysis

	1	1	3	4	5
Cash inflow	$5,000	$5,000	$5,000	$5,000	$5,000
Interest expense	450	360	270	180	90
Taxes	1,365	1,392	1,419	1,446	1,473
Cash flow after taxes and interest	3,185	3,248	3,311	3,374	3,437
Principal	900	900	900	900	900
Cash return to equity	2,285	2,348	2,411	2,474	2,537

The net present value of the investment at period t=0, using the RTE approach is;

$$NPV(R) = \frac{\$2,285}{(1.2)} + \frac{\$2,348}{(1.2)^2} + \frac{\$2,411}{(1.2)^3} + \frac{\$2,474}{(1.2)^4} + \frac{\$2,537}{(1.2)^5} - \$4,500$$

$$NPV(R) = \$2,643.$$

Using the WACC approach, the net present value of the project is;

$$NPV(W) = \sum_{t=1}^{5} \frac{\$5,000(1-.3)}{(1.135)} - \$9,000$$

$$NPV(W) = \$3,162.$$

As in the single-period case, the difference between the two net present values arises because of the different assumptions about debt capacity. Under the WACC approach, debt capacity remains proportional to the market value of the investment throughout its useful life. Thus, k_w will equal .135 in all periods. The value of $k_w^{'}$, on the other hand, varies over the life of the investment. This is shown in the table below where the market values of debt and equity are calculated as of the beginning of each succeeding period of the investment's life.

Table 3. Market Value of Investment at the Beginning of Period

	1	2	3	4	5
Debt Value	\$4,500	\$3,600	\$2,700	\$1,800	\$ 900
Equity Value	7,143	6,286	5,195	3,823	2,114
Total Value	11,643	9,886	7,895	5,623	3,014
D/V	.3864	.3642	.3420	.3201	.2986
$k_w^{'}$	.1498	.1527	.1555	.1584	.1612

As the debt is amortized, the relative contributions of debt and equity to the financing of the investment will change and with them, the value of $k_w^{'}$, the implied weighted average cost of capital. Given the amortization schedule of the example, the relative contribution of equity capital becomes larger in the latter periods of the investment as the leverage ratio declines. Because the cost of equity capital is greater than the cost of debt capital, the weighted average cost of capital likewise becomes larger.

Conclusions

In the one-period case, NPV(W) can be reconciled with NPV(R) by adjusting the debt capacity embedded in the WACC approach to reflect the ratio of incremental debt, in dollars, to the reproduction value of the investment. In the multiperiod case, reconciliation requires two adjustments to the weighted average cost of capital each period; first, the one just described and second, an adjustment to reflect the amortization schedule assumed in the RTE approach.

In deciding whether the analysis supports a preference for one approach over the other, two points are noteworthy. First, to the degree that the incremental investment has a positive net present value, the RTE approach will understate that net present value relative to the WACC approach because the debt capacity implied by the RTE approach, $\Delta D/\Delta I$, declines, other things being equal, with increases in net present value. Debt capacity should more appropriately be defined ex ante, as $\Delta D/\Delta I$. If the investment produces a positive net present value, then debt capacity, ex post, can be said to have increased.

Second, as debt is amortized in the multiperiod case, the RTE approach implies that a decline also occurs in debt capacity (if net present value remains positive). Debt capacity is normally defined as a proportion of equity. As debt is amortized, it is probably more accurate to say that the allocation of debt capacity to debt and to reserve credit has changed. It may also be more accurate to adjust k_e, the equity capitalization rate, to reflect the reallocation.

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